Attendance Problems. Write each fraction as a percent.

1. $\frac{1}{4}$ **2.** $\frac{2}{3}$ **3.** $\frac{3}{8}$ **4.** $\frac{12}{12}$

Evaluate.

5. ${}_6P_3$ **6.** ${}_5P_2$ **7.** ${}_7C_4$ **8.** ${}_8C_6$

- I can find the theoretical probability of an event.
- I can find the experimental probability of an event.

Vocabulary		
probability	outcome	sample space
event	equally likely outcomes	favorable outcomes
theoretical probability	complement	geometric probability
experiment	trial	experimental probability

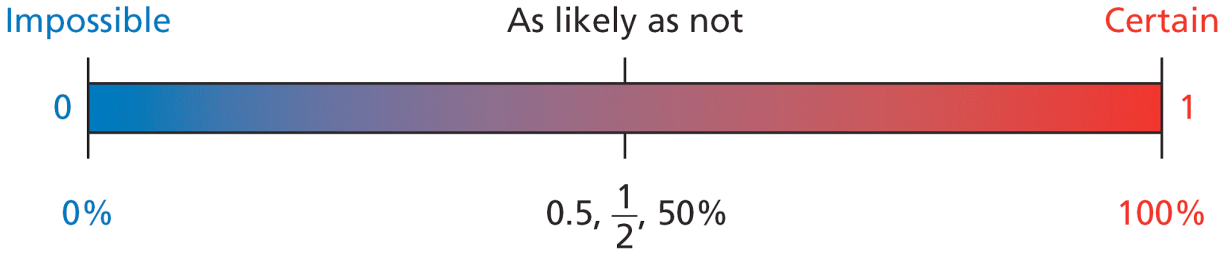
Common Core

CC.9-12.S.MD.7 (+) Analyze decisions and strategies using probability concepts.
CC.9-12.S.CP.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems.

Probability is the measure of how likely an event is to occur. Each possible result of a probability experiment or situation is an **outcome**. The **sample space** is the set of all possible outcomes. An **event** is an outcome or set of outcomes.

Experiment or Situation	Rolling a number cube	Spinning a spinner
		
Sample Space	{1, 2, 3, 4, 5, 6}	{red, blue, green, yellow}

Probabilities are written as fractions or decimals from 0 to 1, or as percents from 0% to 100%.



Equally likely outcomes have the same chance of occurring. When you toss a fair coin, heads and tails are equally likely outcomes. **Favorable outcomes** are outcomes in a specified event. For equally likely outcomes, the **theoretical probability** of an event is the ratio of the number of favorable outcomes to the total number of outcomes.

Theoretical Probability

For equally likely outcomes,

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of outcomes in the sample space}}.$$

Video Example 1

A.

There are 6 hardback books and 11 paperback books on a shelf. What is the probability that a randomly selected book is a paperback?

B.

Two number cubes are rolled. If all numbers are equally likely, what is the probability that the sum is 9?

1 Finding Theoretical Probability

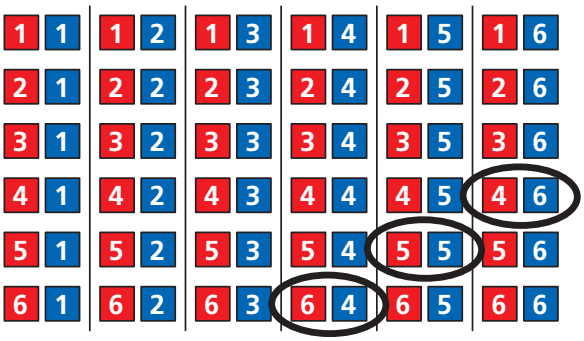
A **A CD has 5 upbeat dance songs and 7 slow ballads. What is the probability that a randomly selected song is an upbeat dance song?**

There are 12 possible outcomes and 5 favorable outcomes.

$$P(\text{upbeat dance song}) = \frac{5}{12} \approx 41.7\%$$

B **A red number cube and a blue number cube are rolled. If all numbers are equally likely, what is the probability that the sum is 10?**

There are 36 possible outcomes.



$$P(\text{sum is } 10) = \frac{\text{number of outcomes with sum of } 10}{36}$$

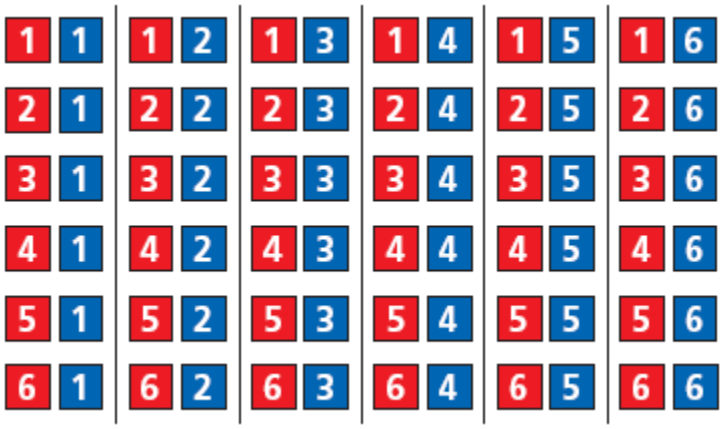
$$P(\text{sum is } 10) = \frac{3}{36} = \frac{1}{12}$$

3 outcomes with a sum of 10: (4, 6) (5, 5), and (6, 4)

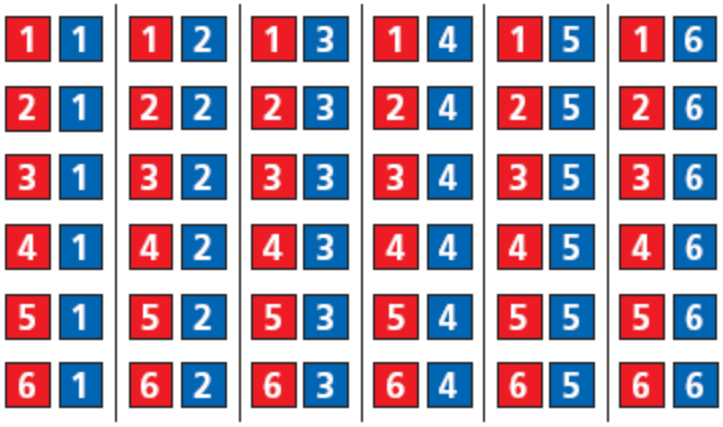
Example 1.

A. Each letter of the word PROBABLE is written on a separate card. The cards are placed face down and mixed up. What is the probability that a randomly selected card has a consonant?

B. Two number cubes are rolled. What is the probability that the difference between the two numbers is 4?



<u>Guided Practice</u>. A red number cube and a blue number cube are rolled. If all numbers are equally likely, what is the probability of the event?



9. The sum is 6.

10. The difference is 6.

11. The red cube is greater.

The sum of all probabilities in the sample space is 1. The **<u>complement</u>** of an event E is the set of all outcomes in the sample space that are not in E.

Complement

The probability of the complement of event E is

$$P(\text{not } E) = 1 - P(E).$$

Video Example 2

A floor is covered with 50 square foot tiles. All the tiles are purple except for those listed in the table below. If you toss a quarter onto the floor at random, what is the probability that it will land on a purple tile?

Floor Tiles	
Color	Number of Tiles
Blue	4
Green	1
Red	2
Yellow	3

2 ***Entertainment Application***

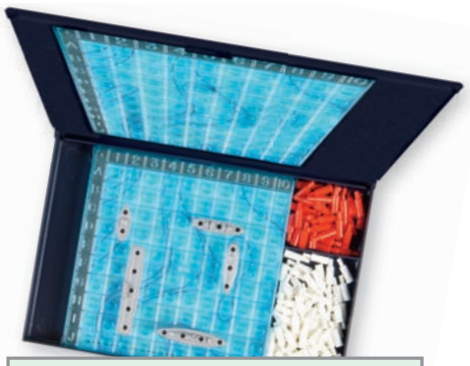
The game Battleship is played with 5 ships on a 100-hole grid. Players try to guess the locations of their opponent's ships and sink them. At the start of the game, what is the probability that the first shot misses all targets?

$$P(\text{miss}) = 1 - P(\text{hit})$$ *Use the complement.*

$$P(\text{miss}) = 1 - \frac{17}{100}$$ *There are 17 total holes covered by game pieces.*

$$= \frac{83}{100}, \text{ or } 83\%$$

There is an 83% chance of the first shot missing all targets.



Battleship Pieces	
Game Piece	Number of Holes Covered
Destroyer	2
Cruiser	3
Submarine	3
Battleship	4
Carrier	5

Example 2. There are 25 students in study hall. The table shows the number of students who are studying a foreign language. What is the probability that a randomly selected student is not studying a foreign language?

Language	Number
French	6
Spanish	12
Japanese	3

12. Guided Practice. Two integers from 1 to 10 are randomly selected. The same number may be chosen twice. What is the probability that both numbers are less than 9?

Video Example 3.

When reserving a cabin at Lost Parrot Cabins, the customer receives a 3 letter confirmation code, with no letter repeated. What is the probability that the customer will receive a code of 3 consecutive vowels (AEIOU)?

3 Finding Probability with Permutations or Combinations

Each student received a 4-digit code to use the library computers, with no digit repeated. Manu received the code 7654. What was the probability that he would receive a code of consecutive numbers?

Step 1 Determine whether the code is a permutation or a combination.

Order is important, so it is a permutation.

Step 2 Find the number of outcomes in the sample space.

The sample space is the number of permutations of 4 of 10 digits.

$${}_{10}P_4 = \frac{10!}{6!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot \cancel{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}{\cancel{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}} = 5040$$

Step 3 Find the favorable outcomes.

The favorable outcomes are the codes 0123, 1234, 2345, 3456, 4567, 5678, 6789, and the reverse of each of these numbers. There are 14 favorable outcomes.

Step 4 Find the probability.

$$P(\text{consecutive numbers}) = \frac{14}{5040} = \frac{1}{360}$$

The probability that Manu would receive a code of consecutive numbers was $\frac{1}{360}$.

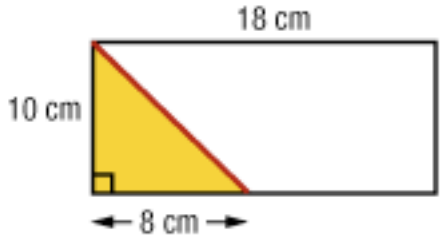
Example 3. Each student receives a 5-digit locker combination. What is the probability of receiving a combination with all odd digits?

13. Guided Practice: A DJ randomly selects 2 of 8 ads to play before her show. Two of the ads are by a local retailer. What is the probability that she will play both of the retailer's ads before her show?

Geometric probability is a form of theoretical probability determined by a ratio of lengths, areas, or volumes.

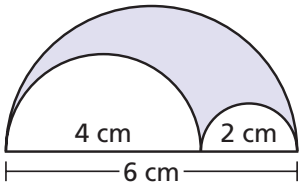
Video Example 4:

A right triangle is drawn inside a rectangle. What is the probability that a randomly chosen point is inside the shaded region?



4 Finding Geometric Probability

Three semicircles with diameters 2, 4, and 6 cm are arranged as shown in the figure. If a point inside the figure is chosen at random, what is the probability that the point is inside the shaded region?



Find the ratio of the area of the shaded region to the area of the entire semicircle. The area of a semicircle is $\frac{1}{2}\pi r^2$.

First, find the area of the entire semicircle.

$A_t = \frac{1}{2}\pi(3^2) = 4.5\pi$ *Total area of largest semicircle*

Next, find the unshaded area.

$A_u = \left[\frac{1}{2}\pi(2^2)\right] + \left[\frac{1}{2}\pi(1^2)\right] = 2\pi + 0.5\pi = 2.5\pi$ *Sum of areas of the unshaded semicircles*

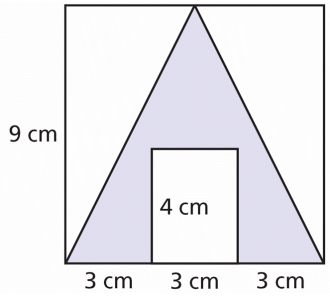
Subtract to find the shaded area.

$A_s = 4.5\pi - 2.5\pi = 2\pi$ *Area of shaded region*

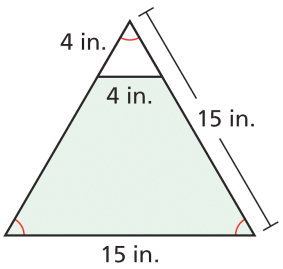
$\frac{A_s}{A_t} = \frac{2\pi}{4.5\pi} = \frac{2}{4.5} = \frac{4}{9}$ *Ratio of shaded region to total area*

The probability that the point is in the shaded region is $\frac{4}{9}$.

Example 4. A figure is created placing a rectangle inside a triangle inside a square as shown. If a point inside the figure is chosen at random, what is the probability that the point is inside the shaded region?



14. <u>Guided Practice</u>. Find the probability that a point chosen at random inside the large triangle is in the small triangle.



You can estimate the probability of an event by using data, or by **<u>experiment</u>**. For example, if a doctor states that an operation "has an 80% probability of success," 80% is an estimate of probability based on similar case histories.

Each repetition of an experiment is a **<u>trial</u>**. The sample space of an experiment is the set of all possible outcomes. The **<u>experimental probability</u>** of an event is the ratio of the number of times that the event occurs, the *frequency*, to the number of trials.

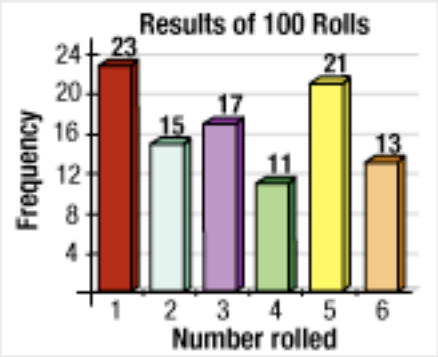
Experimental Probability

$$\text{experimental probability} = \frac{\text{number of times the event occurs}}{\text{number of trials}}$$

Experimental probability is often used to estimate theoretical probability and to make predictions.

Video Example 5:

The bar graph shows the results of 100 tosses of a fair number cube. Find each experimental probability.

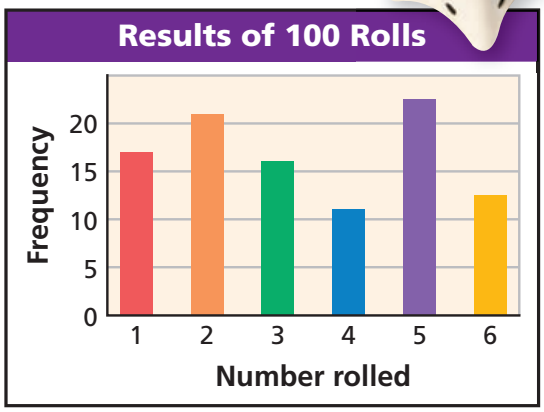


A. Rolling a 5.

B. Rolling a multiple of 3.

5 Finding Experimental Probability

The bar graph shows the results of 100 tosses of an oddly shaped number cube. Find each experimental probability.



A **rolling a 3**

The outcome 3 occurred 16 times out of 100 trials.

$$P(3) = \frac{16}{100} = \frac{4}{25} = 0.16$$

B **rolling a perfect square**

$$P(\text{perfect square}) = \frac{17 + 11}{100}$$

$$= \frac{28}{100} = \frac{7}{25} = 0.28$$

The numbers 1 and 4 are perfect squares. 1 occurred 17 times and 4 occurred 11 times.

C **rolling a number other than 5**

Use the complement.

$$P(5) = \frac{22}{100}$$

5 occurred 22 times out of 100 trials.

$$1 - P(5) = 1 - \frac{22}{100} = \frac{78}{100} = \frac{39}{50} = 0.78$$

<u>Example 5</u>. The table shows the results of a spinner experiment. Find the experimental probability.

Number	Occurrences
1	6
2	11
3	19
4	14

A. Spinning a 4.

B. Spinning a number greater than 2.

<u>Guided Practice</u>. The table shows the results of choosing one card from a deck of cards, recording the suit, and then replacing the card.

Card Suit	Hearts	Diamonds	Clubs	Spades
Number	5	9	7	5

15. Find the experimental probability of choosing a diamond.

16. Find the experimental probability of choosing a card that is not a club**.**

13-2 Theoretical and Experimental Probability *(pp 883-884)* 14-21, 26, 31, 33, 35.